Problem 1
Suppose $f : [a, b] \to \mathbb{R}$ is integrable. Show that for any $c \in \mathbb{R}$,

$$\int_{a+c}^{b+c} f(x - c)\, dx$$

is defined (the underlying function is integrable), and

$$\int_{a+c}^{b+c} f(x - c)\, dx = \int_a^b f(x)\, dx.$$

Problem 2
Suppose $f : [a, b] \to \mathbb{R}$ is bounded and monotone increasing. Show f is integrable.

Problem 3

1. Show that $f : [1, a] \to \mathbb{R}$, $f(x) = \frac{1}{x}$ is integrable, for any $a > 1$.
2. Show that for any $a > 1$ and $b > 0$,

$$\int_1^a \frac{1}{t}\, dt = \int_b^{ab} \frac{1}{t}\, dt.$$

Hint: Scale an arbitrary partition $[x_0, x_1, \ldots, x_n]$ *of* $[1, a]$ *into the partition* $[bx_0, bx_1, \ldots, bx_n]$ *of* $[b, ab]$.

Problem 4
Suppose f is integrable on $[a, b]$. Prove that there is a number x in $[a, b]$ such that

$$\int_a^x f = \int_x^b f.$$

Show by example that it is not always possible to choose $x \in (a, b)$ (i.e. x might have to be on the boundary).

Problem 5
Suppose f is integrable on $[a, b]$. Show that the function $F : [a, b] \to \mathbb{R}, x \mapsto \int_a^x f$ is continuous.

Problem 6
Suppose f is integrable on $[a, b]$. Prove that there is a number x in $[a, b]$ such that

$$\int_a^x f = \int_x^b f.$$

Show by example that it is not always possible to choose $x \in (a, b)$ (i.e. x might have to be on the boundary).